# AN APPLICATION AND COMPARISON OF STATIC MARGINAL ANALYSIS AND GENERALIZED LAGRANGE MULTIPLIERS IN GENERATING A U. S. NAVY REPAIR MATERIALS REQUIREMENT LIST

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# NAVAL POSTGRADUATE SCHOOL
## Monterey, California

# THESIS

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IN GENERATING A U.S. NAVY
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by

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December 1976

Thesis Advisor: A.W. McMasters

## REPORT DOCUMENTATION PAGE

READ INSTRUCTIONS BEFORE COMPLETING FORM

1. REPORT NUMBER
2. GOVT ACCESSION NO.
3. RECIPIENT'S CATALOG NUMBER

4. TITLE (and Subtitle) An Application and Comparison of Static Marginal Analysis and Generalized Lagrange Multipliers in Generating a U.S. Navy Repair Materials Requirement List

5. TYPE OF REPORT & PERIOD COVERED

6. PERFORMING ORG. REPORT NUMBER

7. AUTHOR(s)

Jan Dyhr Christensen

8. CONTRACT OR GRANT NUMBER(s)

9. PERFORMING ORGANIZATION NAME AND ADDRESS

Naval Postgraduate School
Monterey, California 93940

10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS

11. CONTROLLING OFFICE NAME AND ADDRESS

Naval Postgraduate School
Monterey, California 93940

12. REPORT DATE
December 1976

13. NUMBER OF PAGES
42

14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)

15. SECURITY CLASS. (of this report)
Unclassified

15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Lagrange Multipliers
Static Marginal Analysis
Repairables
Stockout
Optimization
Normal Approximation

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

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(20. ABSTRACT Continued)

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Submitted in partial fulfillment of the
requirements for the degrees of

MASTER OF SCIENCE IN MANAGEMENT

and

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
December 1976

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# I. BACKGROUND

The Naval Air Systems Command Instruction 4700.5B of April 30, 1975 is the most recent in a series of instructions defining policy and prescribing procedures for supply support in commercial rework of aeronautical weapon systems and aircraft engines. The implementation of this series of instructions is through the Single Supply Support Control Point (SSSCP) concept. This concept involves and organization, referred to as the SSSCP, which is charged with two objectives of interest to this thesis: first, to achieve dollar savings by providing available government furnished material (GFM) to the commercial contractor for the support of a rework program and secondly, to minimize the rework turnaround time by reducing the overall supply response time through dedicated single point management.

Upon award of a commercial rework contract, an initial supply of available GFM is provided the contractor. The quantity of material provided is determined using a Repair Material Requirements List (RMRL). The RMRL is used by the SSSCP and the contractor as a guide for positioning and requisitioning GFM, respectively, to support an initial 90 day rework production schedule of end items. Timely receipt of this material insures support for the end items first inducted for rework and allows for an orderly implementation of follow-on material support procedures.

Before the development of the RMRL in the early sixties, a contractor was provided 100% of requirements of each line item for each end item to be reworked in the first 90 days of the contract. As an example, if the end item contained ten units of line item Y and 36 end items were to be reworked in the first 90 days, then 36 x 10 or 360 units of issue of item Y would be provided. During the contract performance phase, the contractor was charged to maintain a moving average of the usage rate of each line item and to use this information to order the expected demand for the next increment of end items to be reworked under the contract. The information gathered was subsequently formalized into the current Usage and Assets Report which gives the number of end items reworked and quantities of each line item used since the time of the last report and the quantity of each line item on hand at the time of the report.

By accumulating these records over several contracts the SSSCP was able to devise a replacement factor for each line item, according to the following formula:

$$R_i = \frac{U_i}{Q_i \cdot N_c} \quad ; \qquad i = 1,2,\ldots,n \quad ,$$

where $R_i$ = the replacement factor for the $i^{th}$ line item.

$U_i$ = the total number of line item i used over the several contracts.

$Q_i$ = the quantity of line item i required for each end item.

$N_c$ = the total number of end items requiring item i completed over the several contracts.

n = the total number of different line items applicable to the particular end item.

The resultant $R_i$ is expressed as a percentage and rounded to the nearest integer value. Items with historical usages too low to produce a $R_i$ of 1% or greater after rounding are not included in the RMRL. The combination of the quantity required per end item and the historical demand resulting in such a low $R_i$, apparently does not warrant the inclusion of these items in an initial inventory.

The replacement factors that are 1% or greater after rounding become the key elements in the generation of the RMRL. As presently structured, the RMRL is a computer-based listing giving National Item Identification Number (NIIN)/Manufacture Part Number, nomenclature, unit of issue, number of units of issue required per end item ($Q_i$), replacement factor ($R_i$), gross requirement (explained below), unit of issue cost, cost of the gross requirement and total cost for the RMRL. The gross requirement ($G_i$) is the quantity to initially be shipped to the contractor. It is determined from the quantity required ($Q_i$) per end item and the replacement factor ($R_i$), as follows:

$$G_i = \frac{R_i}{100} \times Q_i \times N \quad ; \quad i = 1,2,\ldots,n$$

where $R_i$ is expressed as a percentage

N = the estimated number of end items to be reworked during the initial 90 days.

n = the number of different line items on the particular RMRL.

It should be noted that $G_i$ is rounded to the next higher integer value and that $G_i$ is never less than one.

The SSSCP, through the RMRL, will provide a contractor with the quantities calculated according to the above formulae as material for initial support. These quantities are the nearest integer value above the mean historical usage as long as the replacement factor, after rounding, is at least 1%. The occasional demand for an item not provided via the RMRL is satisfied by the follow-on material support procedures instituted at the time of contract award.

In an earlier time when there was much less concern over the allocation of limited budgets, the RMRL would not have been required. By providing 100% of engineering requirements, the disruption and cost associated with a stockout and with an order placement could be kept to a minimum during the first 90 days. Of course the amount of funds required to provide inventory storage, protection and control would be high and excessive funds would be spent shipping the very low usage material to one contractor after another until they are finally incorporated in the project or discarded due to wear and tear.

Today, however, with the multitude of military programs vying for a limited budget, a continuing search for cost-saving efficiencies is being carried out at all levels. The RMRL is an example of just such an efficiency, for it provides a much more realistic level of inventory (the

expected demand for 90 days) than was provided prior to the implementation. It should be noted, however, that the present generation technique does not consider any budget constraint as such. The budget consumed is simply the cost of an item times the expected demand for 90 days summed over all items included in the RMRL.

However, because further improvements appeared possible for the RMRL generation technique, an analysis was recently conducted and reported in [1]. The problem addressed in that paper can be stated as follows:

"Given a probability distribution of demand, develop a RMRL generation technique that minimizes the total expected cost of stockouts over all items during the initial contract period, subject to a budget constraint."

If $s_i$ represents the number of units of item i to be stocked initially, then the problem can be stated mathematically as:

Find the value of $s_i \geq 0$ , $i = 1,2,\ldots,n$, which

$$\text{minimizes} \quad \sum_{i=1}^{n} \pi_i \sum_{x=s_i}^{\infty} (x - s_i)\, p_i(x) \qquad (1)$$

$$\text{subject to} \quad \sum_{i=1}^{n} c_i s_i \leq C$$

where $n$ = the number of different line items

$c_i$ = the unit cost for the $i^{th}$ item

$x$ = the demand for a line item

$p_i(x)$ = the probability that x units of line item i will be demanded

$\pi_i$ = the weight (penalty cost or essentiality) of a stockout for item i.

As was noted in [1], one of the problems associated with providing an initial inventory is the lack of knowledge concerning the underlying demand generation probability distribution function. This lack of knowledge usually leads to the use of an assumed distribution or to an inventory based on expected values (the present RMRL approach). To be more specific relative to demand generation a record of demand data for a recently completed contract for the overhaul of 167 R3350 engines was obtained from SSSCP. Demand data for a sample of 200 items out of a total of 2106 items was analyzed. Under the assumption that all items follow the same type of distribution, the Poisson distribution was found to provide the best description of the actual demand data (see Table VII of [1]).[1]

Reference [1] proposed static marginal analysis as a solution procedure for (1). The notion of marginal analysis is that the efficient mix of productive inputs is the mix for which the "marginal product equals marginal costs". In [1] that meant that the composition of the RMRL should be

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[1]Historical mean demands for the individual items were used as the Poisson parameters and simulated demands were compared with the actual usage on the completed contract.

such that the inclusion of an additional unit of an item is solely dependent on the decrease in expected stockout cost per budget dollar consumed. This was mathematically expressed as

$$\frac{\pi_i \, p_i(s_i)}{c_i} \quad ,$$

where $P_i(s_i)$ is the probability that $s_i$ units are used.

The marginal analysis procedure progressively assigns a unit to the inventory of that item which yields the greatest reduction in expected stockout cost per unit increase in budget usage. The first step is to set all $s_i = 0$ and compute

$$\max_i \left\{ \frac{\pi_i}{c_i} \; P_i(s_i + 1) \right\} = \max_i \left\{ \frac{\pi_i}{c_i} \; P_i(1) \right\} . \qquad (2)$$

If the maximum is taken on for item j, set $s_j = 1$ and deduct the unit price $c_j$ from the budget C. This process will continue, using the generalization of (2), as follows:

$$\max \left\{ \max_{i \neq j} \left\{ \frac{\pi_i}{c_i} \; P_i(s_i) \right\} , \; \frac{\pi_j}{c_j} \; P_j(s_j + 1) \right\} , \qquad (3)$$

until adding an additional unit of item i would exceed the budget constraint.

This marginal analysis technique was applied in [1] to the random sample of 200 line items (with $\pi_i = 1$ for all items) in order to generate an RMRL. This RMRL was then

compared with the RMRL generated for the new contract by SSSCP using the current procedures through a simulation of the repair of 167 R3350 engines. It was found that marginal analysis provided reductions of 40% in total number of stockouts and 26% in total number of orders during the rework of the engines over the current procedure for the 200 items. Although improvements were observed, the algorithm used did not produce optimal solutions since, as mentioned in [1],

1. Static marginal analysis is a heuristic process that, by itself, does not guarantee optimality. In particular, the algorithm might stop too soon. If the item $i$ selected from the marginal analysis has a $c_i$ value greater than the remaining budget, the procedure terminates even though some other item j may have a $c_j$ value less than the remaining budget.
2. Due to computer limitations in calculating "powers of e" outside the range -180.218 to +174.673, twenty-three line items were excluded from the marginal analysis and included in the RMRL with the number of items calculated according to current procedures.
3. Severe computer rounding errors occurred when the incremental protection obtained by adding one more item was very small for all line items even though double precision was used.

# II. SOLUTION PROCEDURES

## A. MARGINAL ANALYSIS, POISSON DEMAND WITH NORMAL APPROXIMATION FOR $\lambda > 15$

In order to circumvent some of the problems and limitations described in Chapter I, several actions have to be taken.

To overcome the problem of the inability to calculate "powers of e" outisde the range -180.218 to +174.673 (further limited in [1] to the range ±150.0) the application of a Normal approximation to the Poisson distribution for high mean demands was used in (3). Under Poisson demands (3) takes the following explicit form

$$\max\left\{\max_{i\neq j}\left\{\frac{\pi_i}{c_i}\cdot\frac{e^{-\lambda_i}\lambda_i^{s_i}}{s_i!}\right\},\ \frac{\pi_j}{c_j}\cdot\frac{e^{-\lambda_j}\lambda_j^{s_j+1}}{(s_j+1)!}\right\}. \tag{4}$$

The theory justifying the use of the Normal approximation to the Poisson distribution for high mean values is well known [2,3,4]. The approximation equation is:

$$P_i(s_i) = e^{-\lambda}\frac{\lambda^{s_i}}{s_i!} \doteq \Phi\left(\frac{s_i+\frac{1}{2}-\lambda}{\sqrt{\lambda}}\right) - \Phi\left(\frac{s_i-\frac{1}{2}-\lambda}{\sqrt{\lambda}}\right) \tag{5}$$

Equation (5) can then be used in Equation (4) for $\lambda > 15$. selection of the value 15 as appropriate mean demand for (5) was a result of several test runs in which different values of $\lambda$ was used. These runs showed that for $\lambda > 15$

the resulting inventory vector did not change significantly, while for lower values of $\lambda$ significant changes occured. With a cutoff value of 15, 36% or 72 items would use Equation (5).

The problem of not exhausting the budget C completely can easily be corrected by including a "clean up" algorithm. If the optimal item j selected by the marginal analysis has a $c_i$ > remaining budget then use marginal analysis to select the optimal item i from the set of all items having $c_j \leq$ remaining budget. If selection of items continues in this manner, total budget exhaustion is guaranteed in this case. The approach of static marginal analysis with the "clean up" algorithm, Poisson demands for $\lambda \leq 15$, and Normal approximation to demand for items having $\lambda > 15$ is called the "P,N" procedure in the remainder of this thesis.

The last problem, the problem of rounding errors cannot, at present, be circumvented if the technique of static marginal analysis is used. However, the technique of Generalized Lagrange Multipliers (GLM) might circumvent this problem and will be discussed next.

## B. GENERALIZED LAGRANGE MULTIPLIER APPROACH ASSUMING POISSON DEMAND RATE FOR $\lambda \leq 15$; OTHERWISE ASSUMING NORMAL

Lagrange multipliers are usually introduced in the context of differentiable functions, and are used to produce constrained stationary points. The validity normally appears to be connected with differentiation of the function to be

optimized. However, most real-world problems (e.g. the present multi-item inventory problem) involve discontinuous functions which are to be optimized subject to constraints.

It has been shown [5] that with another viewpoint the use of Lagrange multipliers constitutes a technique whose goal is maximization — rather than location of a stationary point — of a function with constraints, and that in this light there are no restrictions such as continuity or differentiability on the function itself.

Let us suppose there is a set S that is interpreted as the set of possible combinations of items in an inventory. Defined on this strategy set is a real valued payoff function H where H(s) is the payoff obtained by employing the strategy vector $s \varepsilon S$. In addition there are n real valued functions $c_i$, $i = 1,2,...,n$ defined on S, which are called resource functions. The interpretation of $c_i$ is that the employment of strategy vector $s \varepsilon S$ will require $c_i(s)$ of the $i^{th}$ resource. The objective is then to maximize the payoff (or minimize a penalty function) subject to a resource constraint on each resource.

Now recall the inventory problem at hand. We want to minimize the penalty $\pi_i$ resulting from a stockout of the $i^{th}$ item in a situation where the total resource expenditure over all items is subject to a constraint C. Let $s_i$ be the inventory position after the initial RMRL is generated and let $x_i$ be the demand for item i in the initial 90 days

period (the RMRL is intended to cover demand in an initial 90 days period).

The expected number of fulfilled demands for item i is then

$$\sum_{x_i=0}^{s_i} x_i \; P_i(X_i = x_i) + s_i \; P_i(X_i > s_i), \tag{6}$$

which is equivalent to

$$E(X_i) - \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) \; P_i(X_i = x_i). \tag{7}$$

Then, when we try to minimize the expected penalty incurred, or, equivalently to maximize the expected penalty avoided, we get the objective function:

$$\text{maximize } Z(s) =$$

$$\sum_{i=1}^{n} \pi_i [E(X_i) - \sum_{x_i=s_i+1}^{\infty} (x_i - s_i) P_i(X_i = x_i)] \tag{8}$$

Therefore (1) can be rewritten as:

$$\text{maximize } Z(s)$$

$$\text{subject to } \sum_{i=1}^{n} c_i s_i \leq C \tag{9}$$

In a GLM context the problem on hand can be formulated as:

$$\text{maximize } L(s,\theta) = Z(s) - \theta[(\sum_{i=1}^{n} c_i s_i) - C] \qquad (10)$$

where the vector $s \in S$, and $\theta \geq 0$.

Problem (10) is obviously the Lagrangian problem associated with (9). From Everett [5], we know that if a vector S solves (10), then it also solves (9). Guidance on how to adjust $\theta$ in the event that $\sum_i c_i s_i - C \neq 0$ can be obtained from Everett's second theorem. This theorem states that, given two solutions produced by the Lagrange multipliers technique for which only one resource expenditure differs, the ratio of the change in optimum payoff to the change in that resource expenditure is bounded between the two multiplier values that correspond to the changed resource. Let $\theta^1$ and $\theta^2$ be two values of $\theta$ that produce solutions $S_1^*(\theta^1)$ and $S_2^*(\theta^2)$. If we assume that the resource expenditures of the two solutions differ only in the $j^{th}$ resource, i.e.

$$c_i(s_1^*) = c_i(s_2^*) \quad \text{for } i \neq j$$

and that

$$c_j(s_1^*) > c_j(s_2^*)$$

then

$$\theta^2 \geq \frac{Z(s_1^*) - Z(s_2^*)}{c_j(s_1^*) - c_j(s_2^*)} \geq \theta^1 .$$

This indicates very simply in which direction to make changes when employing a trial and error method for adjusting $\theta$ in order to achieve some given constraint on the resource. Decreasing the non-negative multiplier value tends to increase the use of the resource, increasing it use less.

An alternative way to look at the above problem is as a separable or cell problem, in which there is a number of independent areas or cells into which the resources may be committed, and for which the overall payoff is the sum of the payoffs from each independent cell. The advantage of having N single variable problems instead of one N variable problem lies especially in the temporary conversion of the constrained problem to a series of unconstrained maximization problems. In the cell problem with constraints on total resource expenditures, the conversion to unconstrained maximization of the Lagrangian function uncouples what was essentially a combinatorial problem into a vastly simpler problem involving independent strategy selection in each cell.

In the context of the inventory problem on hand, it can be restated such that the objective is to find a strategy set, one element for each cell, which maximizes the total payoff, subject to constraint C on the total resource expenditure. The separated Lagrangian function, one for

each line item, then takes the following form [5]:

$$L_i(s_i,\theta) = Z_i(s_i) - \theta c_i(s_i)$$

The separated Lagrangian expression is maximized by utilizing the following theorem [6].

Theorem: Let $S_i$ be the set 0,1,2,...,3. Then $L_i(s_i,\theta)$ is maximized over $s_i \varepsilon S_i$ at the smallest value of $s_i \varepsilon S_i$ which satisfies the inequality

$$\pi_i P_i(x_i > s_i) \leq \theta c_i. \tag{11}$$

If the Lagrangian in each cell has been correctly maximized, then Everett's Theorem 1 [5] guarantees that the result is a global maximum to the overall problem. It should be noted however that, due to the integer nature of the problem, exact equality between resources used and available may be impossible because of so-called "duality-gaps." In the present case duality-gaps can be explained as abrupt discontinuities in the consumed resource levels generated when $\theta$ is continuously varied. Everett [5] suggested that these "gaps" could be filled by comparing inventory vectors $s^*(\theta_1)$ and $s^*(\theta_2)$, one feasible and near-optimal and the other slightly infeasible. By identifying those items whose unit levels change in going from $s^*(\theta_1)$ to $s^*(\theta_2)$, it may be possible to get closer to optimality

by incrementing those items in the near-optimal solution until there is only a very small slack remaining in the constraint. It should be pointed out that such a procedure will be very time consuming in a real-life situation having thousands of items in the inventory vector.

## C. COMPARATIVE EXAMPLES

Let's assume an initial inventory consisting of three items with the following unit costs and mean historical demands ($\lambda$) as shown in Table I, and a budget constraint of \$143.37, determined as the cost of the inventory that would be shipped under the present system. Finally we assume $\pi_1 = \pi_2 = \pi_3 = 1.0$

TABLE I

| | ITEM 1 | ITEM 2 | ITEM 3 |
|---|---|---|---|
| $\lambda$ | 7.2 | 10.8 | 2.52 |
| Cost | \$16.75 | \$0.05 | \$2.94 |

(1) Standard RMRL calculation

If the inventory is provided according to the present procedure as described in Chapter I, the starting inventory would consist of the units shown in Table II. The probability of a stockout is given as a comparative measure for the three methods.

TABLE II

| | ITEM 1 | ITEM 2 | ITEM 3 | COST |
|---|---|---|---|---|
| INVENTORY | 8 | 11 | 3 | $143.37 |
| P(STOCKOUT) | .4075 | .4207 | .3528 | |

(2) Revised RMRL calculation.

Using the approach of marginal analysis described in Chapter II A, the inventory would be calculated according to marginal protection per $ unit cost. The process begins with Table III.

TABLE III

MARGINAL PROTECTION PER $ UNIT COST

| X | ITEM 1 | INCLUDED AS # | ITEM 2 | INCLUDED AS # | ITEM 3 | INCLUDED AS # |
|---|---|---|---|---|---|---|
| 1 | .05968 | 27 | 20.000 | 1 | .32319 | 20 |
| 2 | .05952 | 28 | 19.996 | 2 | .27241 | 21 |
| 3 | .05887 | 29 | 19.976 | 3 | .19619 | 22 |
| 4 | .05717 | 30 | 19.902 | 4 | .12000 | 24 |
| 5 | .05375 | 31 | 19.698 | 5 | .06282 | 26 |
| 6 | .04828 | 32 | 19.250 | 6 | .02853 | 36 |
| 7 | .04099 | 34 | 18.428 | 7 | .01134 | 40 |
| 8 | .03266 | 35 | 17.136 | 8 | .00404 | 44 |
| 9 | .02432 | 37 | 15.360 | 9 | .00129 | 49 |
| 10 | .01691 | 39 | 13.190 | 10 | .00037 | 52 |
| 11 | .01099 | 41 | 10.802 | 11 | .00010 | 54 |
| 12 | .00660 | 43 | 8.414 | 12 | .00003 | 57 |
| 13 | .00380 | 46 | 6.226 | 13 | - | |
| 14 | .00204 | 47 | 4.374 | 14 | - | |
| 15 | .00103 | 50 | 2.920 | 15 | - | |
| 16 | .00048 | 51 | 1.852 | 16 | - | |
| 17 | .00022 | 53 | 1.118 | 17 | - | |
| 18 | .00009 | 55 | .644 | 18 | - | |
| 19 | .00003 | 56 | .354 | 19 | - | |
| 20 | .00001 | 58 | .186 | 23 | - | |
| 21 | - | | .094 | 25 | - | |
| 22 | - | | .046 | 33 | - | |
| 23 | - | | .020 | 38 | - | |
| 24 | - | | .010 | 42 | - | |
| 25 | - | | .004 | 45 | - | |
| 26 | - | | .002 | 48 | - | |
| 27 | - | | - | | | |

Since the budget constraint is \$143.37 all 20 units of item one, 26 units of item 2 and 12 units of item 3 cannot be included, so we must decide on an initial inventory composition that yields the best marginal protection. This is simply done by including units of each of the three items according to the column "Included as (step) #". But when we reach step number 35 and attempt to include the $8^{th}$ unit of item one, we find this is not possible since we have already used \$133.05 of the budget. The inclusion of the $8^{th}$ unit of item one would increase expenditures to \$148.80. We therefore use up the rest of the budget with units of item three. Table IV gives the final inventory.

TABLE IV

| | ITEM 1 | ITEM 2 | ITEM 3 | COST |
|---|---|---|---|---|
| INVENTORY | 7 | 26 | 8 | \$142.07 |
| P(STOCKOUT) | .5470 | 0 | .0038 | |

As can be seen, the budget is not quite exhausted. However, the decrease in P(Stockout) between Tables II and IV is very significant for items two and three. It has increased by approximately 25% for item one.

3. <u>GLM Inventory Composition</u>

Applying the GLM procedure as described in Chapter II B gives the following cell equations for the data provided:

$$P(x_1 > s_1) \leq 16.75\ \theta$$

$$P(x_2 > s_2) \leq 0.05\ \theta$$

$$P(x_3 > s_3) \leq 2.94\ \theta$$

Table V presents the results of the GLM procedure. The resulting solution is $s_1 = 7$, $s_2 = 23$, and $s_3 = 6$. The total budget consumed is $136.04.

TABLE V

| $\theta$ | COST | CELL 1 (ITEM 1) | CELL 2 (ITEM 2) | CELL 3 (ITEM 3) |
|---|---|---|---|---|
| .01 | $242.47 | 13 | 24 | 8 |
| .04 | $152.79 | 8 | 23 | 6 |
| .05 | $119.09 | 6 | 19 | 6 |
| .041 | $136.04 | 7 | 23 | 6 |
| .042 → .046 | $136.04 | 7 | 23 | 6 |
| .048 | $135.99 | 7 | 22 | 6 |

As can be seen from Table V, the results of this example are very insensitive for a wide range of $\theta$ values. It should be noted however, that with an increasing number of items, solutions closer to optimality (i.e., a more complete budget consumption) are to be expected. This expectation is not based on actual knowledge of closing of duality gaps when the number of items increases, but

rather on empirical observation. Further, in the present context with unit prices ranging from a low of $0.01 to a high of $1,770.00, the inclusions of more items, many of which have a very low price, tends to close any gap.

The probability for a stockout in the above example is 0.547 for item 1, 0.036 for item 2 and 0.0254 for item 3.

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NOTE: In Tables IV and V the mean demand for the three items was rounded to the next high integer value to simplify computation. This results in a slight disadvantage for the last two examples when compared to example one.

## III. RESULTS OF THE ANALYSIS

The first step of the analysis involved implementing the Normal approximation for items with mean greater than 15 and a "clean up" routine in the static marginal analysis inventory generation program.

The results of the simulation with inventories generated by the current procedure system (STANDARD), the approach used in [1] (REVISED), and the approaches described in Chapter II (the (P,N) and the GLM procedures) are given in Table VI.

TABLE VI

SIMULATION OF THE REPAIR OF 167 ENGINES

| METHOD | INITIAL BUDGET USED | # OF ORDERS | # OF STOCKOUTS | RESIDUAL VALUE |
|---|---|---|---|---|
| STANDARD | $138,062.63 | 2510 | 165 | $20,406.85 |
| REVISED | $138,061.48 | 1841 | 99 | $24,795.09 |
| P,N | $138,062.62 | 1124 | 50 | $33,789.35 |
| GLM | $138,044.61 | 1125 | 49 | $33,763.40 |

In Table VI above, column 1 (initial budget used) gives the total value of the initial inventory generated by each method. In all cases the budget constraint was $138,062.63, which was the budget used under the present system. Column 2 (# of orders) gives the total number of

orders submitted by the contractor for additional units of the 200 line items during the simulation of the repair of 167 engines. Column 3 gives the total number of stockouts observed during the simulation, and the last column gives the total value of the residual line items in the contractor's inventory after completion of the repair of all 167 engines.

As can be seen from Table VI substantial reductions in both the number of orders and the number of stockouts were obtained by using a Normal demand distribution for high demand items instead of merely their historic mean demands in the development of the initial inventories.

The behavior of the residual value of the inventory remaining at the contractor's facility at the end of the contract is not unexpected. As better protection against stockouts are provided, the probabilities of larger final inventories increases. Unfortunately these inventories must be retrieved by the SSSCP at contract termination time. A reduction in this residual inventory might be obtained by changing the contractor's ordering policy in the late part of an overhaul.

As was shown in the Chapter II example, a trial and error approach is required for the GLM method. This involves assuming a value for $\theta$, determining the associated $s_i$ values from Equation (11) and then the total amount of the budget consumed. Table VII summarizes the analysis.

TABLE VII

LAGRANGE MULTIPLIERS AND THE
ASSOCIATED RMRL COST

| θ VALUE | RMRL COST | BUDGET CONSTRAINT |
|---|---|---|
| .00100 | $160,714.22 | |
| .00200 | $144,330.26 | |
| .00300 | $142,659.47 | |
| .00400 | $140,663.48 | |
| .00410 | $140,552.29 | |
| .00411-13 | $140,204.61 | $138,062.61 |
| .00414 | $137,684.61 | |

As can be seen from Table VII, the optimal solution must be associated with a $\theta$ value between .00413 and .00414; the $\theta$'s between these values were then investigated and the change in the inventory vector was observed. This investigation revealed that there existed one of the previously described duality gaps resulting in the number of units of only one item changing between these two values. The item mean was 440.64 units, its cost was $120.00, and the number of units jumped from 440 units at $\theta = .00414$ to 461 units at any $\theta$ value greater than .00414. A solution, as suggested by Everett, to this gap problem is to apply $\theta = .00414$ and then to include 3 more units of that specific item, leaving a gap of $18.02.

Table VI suggests that for large numbers of items the P,N and GLM procedures gives quite comparable results. Perhaps this should not be too surprising since both methods use the ratio of the form $R_i(s_i)/c_i$ in determining optimal $s_i$ values.

A disadvantage of the P,N procedure is that the inventory vector must be generated in steps, each step requiring comparison of the marginal protection of each of the N items in the vector. In contrast, the GLM procedure can generate the inventory vector in one step if $\theta$ is known. The generation of the inventory vector by the P,N approach required close to two minutes of CPU time on the Naval Postgraduate School IBM 360/67 computer. The GLM procedure solved the problem in a little less than 10 seconds for a single $\theta$ value. It should be noted however that in the GLM approach several runs might be required before the $\theta$ value that gives budget exhaustion or close to budget exhaustion is found. On the other hand, with continued use of the GLM procedure in the context of generating inventory vectors, a priori knowledge of the approximate value of $\theta$ could reduce the number of trial runs considerably.

The potential reduction in computer time by using the GLM method should be even more significant in a real-life situation with tenfold as many items in the inventory vector. This is because the comparison of the marginal protection for each of the items in each step in the P,N approach would result in more than linear growth in CPU time usage, whereas

the GLM procedure would have a close to linear growth. A closer inspection and comparison of the two inventory vectors generated by the P,N and the GLM methods uncovered minor differences in a few line items (under one method the initial inventory position for a given line item was 10 whereas using the second method was 11).

After the above simulations with the original budget constraint were conducted, the sensitivity to changes in the budget constraint C was examined. The results of this sensitivity analysis are given in Table VIII.

TABLE VIII

SENSITIVITY ANALYSIS WITH REGARD TO INITIAL BUDGET CONSTRAINT

| INITIAL BUDGET | REVISED #STOCK-OUTS | REVISED # OF ORDERS | P,N #STOCK-OUTS | P,N # OF ORDERS | GLM #STOCK-OUTS | GLM # OF ORDERS |
|---|---|---|---|---|---|---|
| $138,062.61 | 99 | 1841 | 50 | 1124 | 49 | 1125 |
| $125,000.00 | 102 | 1921 | 51 | 1151 | 50 | 1150 |
| $115,000.00 | 132 | 2031 | 61 | 1231 | 61 | 1230 |
| $105,000.00 | 152 | 2125 | 81 | 1297 | 81 | 1298 |
| $ 95,000.00 | 174 | 2322 | 82 | 1382 | 83 | 1382 |
| $ 85,000.00 | 223 | 2604 | 95 | 1416 | 95 | 1416 |

Table VIII shows that the P,N and the GLM procedures continued to give comparable results and that both performed far better than the REVISED approach as developed in [1] and the current approach (STANDARD) which had 165 stockouts and 2510 orders and required a budget of $138,062.61.

As was noted earlier a budget reduction is not possible in the STANDARD since the generation technique is to have an inventory equal to the mean historic demand regardless of the associated costs. Any reduction would mean a deviation from this technique.

Table VIII shows that even for a budget reduction of approximately 39% the P,N and the GLM procedures will still perform better than REVISED and STANDARD, i.e. will result in less total stockouts and orders. At the 39% budget reduction point REVISED performed comparable to STANDARD at full budget.

It is important to emphasize any savings in the initial budget are only temporary since the total usage over the entire contract will be the same regardless of when the material is delivered. The new generation technique merely allows for a temporary reallocation of the initial fund savings to other programs.

## IV. SUMMARY AND RECOMMENDATIONS

Two improved methods for the generation of RMRL's were developed, employing in the first case static marginal analysis with a "clean up" procedure and, in the second case, the use of a generalized Lagrange multiplier (GLM) approach to a cell problem. The use of both static marginal analysis and GLM requires that the underlying probability distribution for demand be known.

A RMRL giving the initial inventory vector was generated using both techniques and the subsequent demands and ordering during overhaul of 167 R3350 engines were determined using simulation. Historical data [1] suggested that item demand followed the Poisson distribution. For items with a mean historical demand greater than 15, the Normal approximation to the Poisson distribution was used.

The numbers of stockouts and orders were then compared with the numbers resulting from a similar simulation using the present RMRL generation technique and the technique employed in [1]. Both methods reduced the numbers of stockouts and orders. Reductions obtained were 70% in stockouts and 55% in orders when compared with the present technique (based on mean historic demand), and 50% and 39%, respectively, when compared with the approach employed in [1].

The GLM procedure appears to be the more economical of the two methods in terms of computer time usage.

Even though the number of stockouts and the number of orders were drastically reduced, no general claim of optimality can be made. In the case of static marginal analysis the budget was nearly exhausted (1 cent left), but the heuristic nature of the process does not guarantee optimality. The existence of optimal solutions that can be found by the GLM procedure depends upon an approximate concavity requirement (see Everett [5]) in the region of the solution. As was previously discussed, a duality gap was found in the area of interest in the present case, and hence only a feasible, sub-optimal solution was reached. An inspection and comparison of a feasible, sub-optimal inventory vector and a slightly infeasible inventory vector was performed as suggested in references [5] and [6]. This inspection resulted in the inclusion of three more units of a given line item but the final solution was still believed to be sub-optimal. A duality gap remained although it had been reduced.

As mentioned above, previous work had shown that the Poisson distribution was applicable for the R3350 engine overhaul. In any new attempt to apply either the GLM or the static marginal analysis techniques in the generation of an RMRL, the necessary first step is to determine the specific demand distribution.

The large size (value) of the residual inventory after the simulated repair of all 167 engines suggests that a study should be made of the ordering policy after the

determination and delivery of the initial inventory. An ordering policy which reduces these residuals without creating excessive stockouts seems appropriate. The ordering policy used in this context in order to be able to make comparisons of the influence of the generation technique has been that of the SSSCP. According to this policy, the initial RMRL is the basis for the contractor's future orders, i.e. as soon as the inventory drops below a certain level he is allowed to order the difference between the RMRL quantity and what he has on hand minus backorders. When the number of end items left to rework is less than the number used for generating the RMRL (in this case 36 engines) the reorder policy changes in a way such that the reorder quantity limit becomes the expected usage per line item times the number of end items left to rework.

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APPENDIX A

DATA BASE

| NIIN | QTY PER ENG | RPL FCT | UNIT PRICE | CONTRACT USAGE |
|---|---|---|---|---|
| 7047523 | 0 | 0 | 20.00 | 0 |
| 1749497 | 0 | 0 | 2.04 | 35 |
| 916593 | 0 | 0 | 69.00 | 0 |
| 184096 | 1 | 1 | 334.00 | 2 |
| 242896 | 2 | 10 | 16.75 | 32 |
| 316599 | 4 | 1 | 40.00 | 6 |
| 379260 | 9 | 1 | 0.08 | 10 |
| 379363 | 1 | 7 | 5.21 | 11 |
| 379423 | 2 | 13 | 0.10 | 45 |
| 379691 | 2 | 15 | 0.05 | 50 |
| 458865 | 2 | 5 | 0.69 | 17 |
| 1416693 | 1 | 6 | 0.05 | 10 |
| 1711509 | 1 | 7 | 2.94 | 12 |
| 2062981 | 1 | 3 | 0.30 | 5 |
| 2095394 | 1 | 7 | 0.01 | 12 |
| 2131789 | 1 | 4 | 54.05 | 6 |
| 2131813 | 3 | 1 | 107.80 | 7 |
| 2173185 | 1 | 32 | 12.18 | 53 |
| 2440514 | 5 | 2 | 0.10 | 15 |
| 2537554 | 1 | 16 | 0.36 | 27 |
| 2750475 | 7 | 8 | 1.05 | 97 |
| 2762769 | 18 | 1 | 0.05 | 32 |
| 2906984 | 1 | 17 | 0.57 | 28 |
| 2913285 | 1 | 16 | 0.06 | 26 |
| 2913303 | 2 | 10 | 0.04 | 34 |
| 2923120 | 2 | 4 | 0.20 | 12 |
| 2986868 | 2 | 1 | 0.14 | 4 |
| 3036123 | 1 | 2 | 37.43 | 3 |
| 3049019 | 1 | 37 | 35.00 | 61 |
| 3075570 | 2 | 4 | 3.33 | 15 |
| 3102870 | 1 | 1 | 88.75 | 2 |
| 3109004 | 1 | 13 | 86.19 | 21 |
| 3128836 | 1 | 3 | 129.00 | 5 |
| 3133636 | 7 | 5 | 0.47 | 61 |
| 3133653 | 1 | 6 | 36.84 | 10 |
| 3144651 | 7 | 3 | 0.12 | 30 |
| 3236729 | 1 | 7 | 12.00 | 12 |
| 3236730 | 1 | 7 | 5.20 | 11 |
| 3260802 | 1 | 14 | 136.21 | 24 |
| 3266635 | 1 | 28 | 29.00 | 46 |

| NIIN | QTY PER ENG | RPL FCT | UNIT PRICE | CONTRACT USAGE |
|---|---|---|---|---|
| 3266649 | 1 | 1 | 6.05 | 1 |
| 3266652 | 1 | 12 | 9.10 | 20 |
| 3266657 | 1 | 5 | 100.00 | 8 |
| 3320476 | 1 | 7 | 3.00 | 12 |
| 3320477 | 2 | 6 | 21.50 | 20 |
| 3320485 | 1 | 5 | 77.04 | 8 |
| 3354807 | 4 | 3 | 40.00 | 17 |
| 3357073 | 22 | 1 | 0.27 | 20 |
| 3421180 | 1 | 38 | 0.72 | 63 |
| 3441409 | 3 | 6 | 36.50 | 30 |
| 3459562 | 1 | 5 | 6.13 | 8 |
| 3596844 | 1 | 1 | 0.74 | 2 |
| 4423415 | 2 | 7 | 4.58 | 23 |
| 4451522 | 1 | 3 | 129.96 | 5 |
| 4460530 | 1 | 4 | 66.00 | 6 |
| 4788907 | 1 | 34 | 0.49 | 56 |
| 4789077 | 2 | 8 | 0.96 | 26 |
| 4848265 | 2 | 1 | 0.08 | 2 |
| 5063334 | 18 | 1 | 138.20 | 24 |
| 5085494 | 6 | 5 | 19.00 | 55 |
| 5126425 | 1 | 20 | 29.61 | 33 |
| 5129631 | 1 | 7 | 1.23 | 11 |
| 5129635 | 1 | 3 | 37.21 | 5 |
| 5129707 | 1 | 2 | 48.50 | 3 |
| 5129739 | 1 | 2 | 225.00 | 3 |
| 5129777 | 1 | 15 | 111.00 | 25 |
| 5129790 | 1 | 13 | 7.39 | 22 |
| 5150800 | 1 | 8 | 2.88 | 14 |
| 5163785 | 2 | 10 | 4.90 | 35 |
| 5164844 | 1 | 29 | 54.63 | 48 |
| 5255110 | 6 | 6 | 0.06 | 63 |
| 5285683 | 1 | 10 | 0.98 | 17 |
| 5516876 | 4 | 7 | 2.21 | 49 |
| 5555751 | 1 | 25 | 14.00 | 42 |
| 5668943 | 1 | 4 | 592.00 | 6 |
| 5727165 | 1 | 33 | 6.76 | 55 |
| 5739655 | 3 | 5 | 218.00 | 25 |
| 5849563 | 4 | 4 | 0.02 | 30 |
| 5918215 | 1 | 8 | 849.00 | 14 |
| 5941171 | 1 | 5 | 42.43 | 8 |
| 6023691 | 2 | 13 | 144.00 | 42 |
| 6048493 | 8 | 5 | 0.25 | 68 |
| 6048494 | 1 | 6 | 0.25 | 10 |
| 6058293 | 1 | 2 | 78.09 | 3 |
| 6066965 | 2 | 7 | 170.10 | 22 |
| 6182527 | 1 | 14 | 0.21 | 24 |
| 6233794 | 1 | 11 | 10.61 | 19 |
| 6250754 | 1 | 4 | 399.91 | 7 |
| 6322052 | 1 | 5 | 0.02 | 9 |
| 6384070 | 6 | 1 | 1.10 | 9 |

| NIIN | QTY PER ENG | RPL FCT | UNIT PRICE | CONTRACT USAGE |
|---|---|---|---|---|
| 6501192 | 1 | 23 | 50.56 | 39 |
| 6501194 | 1 | 2 | 15.50 | 4 |
| 6514692 | 1 | 1 | 1770.00 | 1 |
| 6547284 | 4 | 4 | 42.37 | 24 |
| 6547287 | 4 | 7 | 55.00 | 46 |
| 6598523 | 1 | 2 | 563.00 | 3 |
| 6622281 | 8 | 1 | 12.00 | 15 |
| 6622476 | 1 | 35 | 0.50 | 58 |
| 6736677 | 1 | 2 | 950.00 | 4 |
| 6969469 | 1 | 8 | 60.00 | 13 |
| 6969477 | 1 | 1 | 8.08 | 2 |
| 6974802 | 4 | 2 | 58.06 | 12 |
| 7047531 | 1 | 3 | 176.00 | 5 |
| 7161469 | 1 | 23 | 29.50 | 39 |
| 7162944 | 1 | 2 | 292.52 | 3 |
| 7162955 | 9 | 2 | 530.00 | 24 |
| 7172218 | 1 | 2 | 146.26 | 4 |
| 7172404 | 4 | 2 | 40.19 | 11 |
| 7204894 | 1 | 15 | 18.98 | 25 |
| 7303275 | 1 | 8 | 99.00 | 14 |
| 7575069 | 1 | 6 | 0.24 | 10 |
| 7974052 | 27 | 1 | 0.08 | 50 |
| 8117017 | 1 | 23 | 10.50 | 38 |
| 8301942 | 1 | 3 | 35.00 | 5 |
| 8303008 | 1 | 2 | 26.80 | 4 |
| 8303010 | 1 | 2 | 48.50 | 3 |
| 8303012 | 1 | 4 | 61.41 | 7 |
| 8303040 | 1 | 7 | 57.83 | 11 |
| 8846264 | 2 | 10 | 775.22 | 32 |
| 8991790 | 1 | 1 | 0.50 | 1 |
| 9038282 | 1 | 10 | 0.03 | 16 |
| 9152018 | 2 | 1 | 3.50 | 3 |
| 9317218 | 2 | 3 | 36.10 | 10 |
| 9631387 | 3 | 13 | 161.95 | 65 |
| 9631388 | 3 | 4 | 152.00 | 22 |
| 9670092 | 1 | 17 | 13.00 | 29 |
| 9773423 | 6 | 3 | 17.50 | 32 |
| 9782993 | 9 | 1 | 8.45 | 17 |
| 330368 | 20 | 12 | 35.30 | 407 |
| 489131 | 45 | 3 | 62.40 | 190 |
| 1006170 | 2 | 102 | 0.86 | 340 |
| 2076434 | 18 | 5 | 0.42 | 157 |
| 2105349 | 9 | 41 | 0.06 | 617 |
| 2250470 | 2 | 51 | 74.00 | 172 |
| 2750632 | 18 | 9 | 0.05 | 263 |
| 2913291 | 4 | 18 | 0.06 | 123 |
| 2978384 | 6 | 11 | 1.32 | 112 |
| 3036014 | 1 | 43 | 2.70 | 71 |
| 3037779 | 1 | 44 | 0.09 | 73 |

| NIIN | QTY PER ENG | RPL FCT | UNIT PRICE | CONTRACT USAGE |
|---|---|---|---|---|
| 3065839 | 4 | 69 | 18.18 | 461 |
| 3108941 | 9 | 14 | 9.60 | 208 |
| 3108946 | 8 | 48 | 2.21 | 637 |
| 3109005 | 6 | 12 | 85.12 | 120 |
| 3133661 | 8 | 7 | 0.16 | 100 |
| 3133672 | 84 | 2 | 15.44 | 243 |
| 3144629 | 3 | 51 | 0.22 | 256 |
| 3144661 | 9 | 16 | 0.34 | 245 |
| 3148139 | 18 | 10 | 1.40 | 302 |
| 3306478 | 16 | 14 | 10.07 | 376 |
| 3306479 | 16 | 9 | 21.97 | 237 |
| 3963912 | 8 | 48 | 0.11 | 646 |
| 4335422 | 2 | 65 | 3.60 | 218 |
| 5058611 | 150 | 1 | 2.99 | 140 |
| 5058634 | 3 | 22 | 854.00 | 108 |
| 5129694 | 2 | 60 | 0.01 | 199 |
| 5129730 | 2 | 89 | 0.03 | 298 |
| 5155558 | 2 | 56 | 0.14 | 187 |
| 5224835 | 2 | 34 | 9.60 | 113 |
| 5277488 | 2 | 78 | 0.15 | 260 |
| 5285651 | 32 | 10 | 0.12 | 510 |
| 5676397 | 6 | 7 | 0.97 | 70 |
| 5796324 | 2 | 23 | 1.48 | 76 |
| 5804444 | 2 | 73 | 1.53 | 245 |
| 5901802 | 150 | 1 | 9.20 | 150 |
| 5995989 | 5 | 44 | 0.06 | 368 |
| 5996406 | 2 | 71 | 0.14 | 238 |
| 6138001 | 168 | 2 | 8.40 | 588 |
| 6563168 | 3 | 60 | 0.01 | 303 |
| 6621800 | 6 | 25 | 10.00 | 249 |
| 6807297 | 5 | 13 | 0.03 | 110 |
| 6807628 | 24 | 2 | 0.59 | 98 |
| 7172223 | 4 | 35 | 0.10 | 235 |
| 7194426 | 2 | 30 | 72.57 | 99 |
| 7197729 | 8 | 30 | 1.25 | 401 |
| 8032651 | 4 | 61 | 0.22 | 410 |
| 9500039 | 118 | 1 | 3.68 | 102 |
| 1187490 | 9 | 75 | 0.10 | 1132 |
| 1476306 | 24 | 21 | 0.10 | 833 |
| 1984735 | 218 | 31 | 0.23 | 11329 |
| 2105221 | 36 | 82 | 0.08 | 4944 |
| 2686041 | 16 | 45 | 0.07 | 1200 |
| 3036651 | 36 | 12 | 0.69 | 727 |
| 3133641 | 9 | 48 | 0.15 | 726 |
| 3439277 | 72 | 92 | 0.05 | 11074 |
| 4790482 | 6 | 111 | 0.28 | 1111 |
| 5159073 | 6 | 85 | 0.16 | 855 |
| 5309323 | 72 | 22 | 0.09 | 2589 |
| 5513093 | 35 | 17 | 0.37 | 977 |

| NIIN | QTY PER ENG | RPL FCT | UNIT PRICE | CONTRACT USAGE |
|---|---|---|---|---|
| 5804634 | 102 | 12 | 120.00 | 1961 |
| 5961865 | 54 | 17 | 0.17 | 1510 |
| 5966095 | 26 | 48 | 0.23 | 2078 |
| 6061829 | 205 | 4 | 1.12 | 1495 |
| 6118234 | 300 | 4 | 12.16 | 1922 |
| 6527000 | 52 | 28 | 0.02 | 2431 |
| 6621790 | 36 | 54 | 0.53 | 3258 |
| 6724938 | 18 | 60 | 0.04 | 1804 |
| 7220101 | 18 | 50 | 0.04 | 1491 |
| 8641347 | 150 | 9 | 6.52 | 2285 |
| 9086292 | 15 | 33 | 0.17 | 820 |
| 2973756 | 0 | 0 | 0.69 | 8001 |

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